

Back action cancellation in resolved sideband regime

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We consider the simple model of measurement of mechanical oscillator position via Fabry-Pero cavity pumped by detuned laser (end mirror of cavity is mass of oscillator) in resolved sideband regime when laser is detuned from cavity's frequency by frequency of mechanical oscillator $\pm\omega_m$ and relaxation rate γ of cavity is small: $\gamma \ll \omega_m$. We demonstrate fluctuation back action cancellation in reflected wave. However, it does not allow to circumvent Standard Quantum Limit, the reason of it is the dynamic back action.

I. INTRODUCTION

Vacuum fluctuations of mechanical oscillator displacement is a key prediction of quantum mechanics which is interesting to verify. Optical parametric cooling of mechanical nano-oscillators close to their ground state [1–10] makes it easier to observe the quantum behavior. An impressive observation of a mesoscopic mechanical oscillator close to its ground state was recently made using the resolved sideband laser cooling [11].

The quantum fluctuations are responsible for quantum back action (disturbance of the quantum system) induced by a measuring device. As for continuous position measurement it results in an accuracy restriction (limitation) known as *Standard Quantum Limit* (SQL) first derived by Braginsky [12, 13]. Observation of back action as well as SQL is difficult to realize for mechanical system. But recently it became possible with the help of opto-mechanical devices that couple optical degrees of freedom to the mechanical oscillator and that way approach the quantum regime [4–6]. Now several groups are close to this goal [7–10, 14].

Usually the resolved sideband regime is used under the conditions of a frequency shift between the laser frequency and the frequency of a cavity mode. This shift equals to the frequency of the mechanical oscillator which is much larger than the optical bandwidth of the cavity mode (2.1). In this paper we analyze this regime in order to find minimal signal force acting on a mechanical oscillator. We show that back action is canceled. However, it does not allow to circumvent SQL due to *dynamical* back action (introduction of damping into mechanical oscillator).

II. SIMPLIFIED MODEL

As the model of an opto-mechanical system we consider a Fabry-Perot cavity with a movable mirror (see Fig. 1). (Note that this model is also valid for interaction of mechanical oscillator with light waves in toroidal microcavities [2–4].) The end mirror of the cavity acts as a mass of a mechanical oscillator of a frequency ω_m and a damping rate γ_m . The cavity is pumped by a laser of frequency ω_L detuned from resonant frequency

ω_0 of the cavity the mode by $\Delta = \omega_0 - \omega_L$. In such opto-mechanical system back action is induced by the fluctuations of the light pressure force. Measuring the amplitude quadrature of the output wave one gets the information about the mechanical displacement. What we are interested in is the the minimal force(acting on the mechanical oscillator)that could be measured.

All further considerations are made under the condition of the resolved sideband regime:

$$\gamma_m \ll \gamma \ll \omega_m, \quad (2.1)$$

where γ is relaxation rate of the cavity mode.

The theoretical background for this model is well known [3, 10, 15, 16]. We start from simplified model considering two cases of positive and negative detuning ($\Delta = \pm\omega_m$) separately.

A. Positive detuning

Let us consider the case of positive detuning $\Delta = \omega_m$ (laser frequency is *smaller* than cavity frequency). There is a conventional set of equations in frequency domain for the wave amplitude $a(\Omega)$ inside the cavity and the Fourier transform $b(\Omega)$ of the mechanical oscillator's annihilation operator [16–19]:

$$(\gamma - i\nu) a + iG_0 b = \sqrt{2\gamma} a_{in}, \quad (2.2a)$$

$$iG_0^* a + (\gamma_m - i\nu) b = \sqrt{2\gamma_m} b_{th} + f, \quad (2.2b)$$

$$a_-^\dagger [\gamma - i(2\omega_m + \nu)] = \sqrt{2\gamma} a_{in-}^\dagger, \quad (2.2c)$$

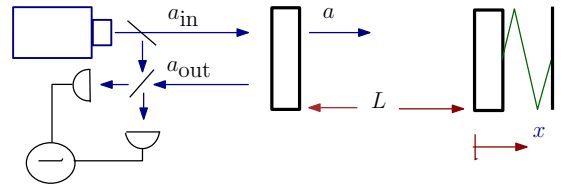


FIG. 1: Fabry-Perot cavity with one movable mirror. Laser is detuned from cavity frequency. The measurement of quadrature in output wave provides information on mirror's displacement.

$$\mathbf{a}_{\text{out}} = -\mathbf{a}_{\text{in}} + \sqrt{2\gamma} \mathbf{a}, \quad (2.2d)$$

$$\mathbf{a} \equiv \mathbf{a}(\Omega), \quad \mathbf{a}_{\text{in}-}^{\dagger} \equiv \mathbf{a}_{\text{in}}^{\dagger}(-\Omega), \quad \Omega \equiv \omega_m + \nu, \\ \mathbf{b} \equiv \mathbf{b}(\Omega), \quad \mathbf{b}_{\text{th}} \equiv \mathbf{b}_{\text{th}}(\Omega), \quad \mathbf{a}_{-}^{\dagger} \equiv \mathbf{a}^{\dagger}(-\Omega) \quad (2.2e)$$

$$|G_0| = \sqrt{\frac{kI_0}{mL\omega_m}}, \quad f = \frac{F(\Omega)}{i\sqrt{\hbar\omega_m m}}. \quad (2.2f)$$

Here \mathbf{a}_{in} and \mathbf{a}_{out} describe the fluctuation amplitudes of the incident and reflected waves, G_0 is an opto-mechanical constant, k is a wave vector of light wave, I_0 is the light power inside the cavity, m is the mass of the movable mirror, L is a mean distance between the mirrors of the cavity, $F(\Omega)$ is a signal force (details in Appendix A).

The main simplification in the set (2.2) is the equation (2.2c) where the interaction between the left sideband amplitudes $\mathbf{a}^{\dagger}(-\Omega)$ and the mechanical oscillator is omitted. This approximation is based on condition (2.1).

We calculate determinant of the set (2.2a, 2.2b):

$$D = (\gamma - i\nu)(\gamma_m + \eta - i\nu), \quad (2.3)$$

$$\eta \equiv \frac{|G_0|^2}{\gamma - i\nu} = \eta_r + i\eta_i, \quad \eta_r \equiv \frac{|G_0|^2\gamma}{\gamma^2 + \nu^2}. \quad (2.4)$$

Here η describes ponderomotive rigidity which transforms into damping in resolved sideband case. Indeed, the real part η_r is *positive* ponderomotive damping (it is this damping that is responsible for parametric cooling [1–10]). Imaginary part η_i is negligibly small ($\eta_i = \eta_r\nu/\gamma \ll \eta_r$) due to condition (2.1). One may easily find solution of (2.2) for $\mathbf{a}(\Omega)$, $\mathbf{a}^{\dagger}(-\Omega)$ and calculate \mathbf{a}_{out}

$$\mathbf{a}_{\text{out}} = \frac{\gamma + i\nu}{\gamma - i\nu} \left(1 - \frac{2\eta_r}{\gamma_m + \eta - i\nu} \right) \mathbf{a}_{\text{in}-} \\ - \frac{iG_0\sqrt{2\eta_r}}{|G_0|(\gamma_m + \eta - i\nu)} \sqrt{\frac{\gamma + i\nu}{\gamma - i\nu}} \left(\sqrt{2\gamma_m} \mathbf{b}_{\text{th}} + f \right), \quad (2.5)$$

$$\mathbf{a}_{\text{out}-} = \frac{\gamma - i(2\omega_m + \nu)}{\gamma + i(2\omega_m + \nu)} \mathbf{a}_{\text{in}-}. \quad (2.6)$$

The second term in round brackets in (2.5) ($\sim \eta_r$) describes back action via light pressure force — introduced damping η_r is proportional to the power I_0 circulating in the cavity. However, it can be shown by straightforward calculation that back action will be completely compensated.

Let us measure the quadrature $y = (\mathbf{a}_{\text{out}}(\Omega) + \mathbf{a}_{\text{out}}^{\dagger}(-\Omega))/\sqrt{2}$ in the output wave. Then the double-sided spectral density S_y of the output quadrature y is as follows:

$$S_y(\Omega) = \frac{1}{2} + \frac{2\eta_r\gamma_m n_T}{|\gamma_m + \eta - i\nu|^2}, \quad (2.7)$$

where n_T is the mean number of the thermal photons in the mechanical oscillator. Formula (2.7) is valid for positive frequencies, however, it is not a problem due to

$S_y(\Omega) = S_y(-\Omega)$. To calculate the spectral density S_y we used conventional correlators (A4).

Well known that the term describing back action noise in the output spectral density is proportional to the squared power I_0 circulating in the cavity — see, for example, [19, 20]. In our notations it corresponds to the term proportional to $\sim \eta_r^2$ — see definitions (2.2f, 2.4). However, we see that back action term $\sim \eta_r^2$ is absent in (2.7), hence, it demonstrates compensation of back action in resolved sideband regime.

Note that the same result may be obtained for *any other* quadrature $y_{\theta} = (\mathbf{a}_{\text{out}}(\Omega)e^{-i\theta} + \mathbf{a}_{\text{out}}^{\dagger}(-\Omega)e^{i\theta})/\sqrt{2}$ due to approximation (2.2c) (which means that the left sideband $\mathbf{a}^{\dagger}(-\Omega)$ does not interact with the mechanical degree of freedom).

This simplified model is correct for the description of the mechanical cooling. Indeed, straightforward calculation of the mechanical oscillator's mean energy gives

$$\mathcal{E}_m = \hbar\omega_m \left(\frac{\gamma_m n_T}{\gamma_m + \eta_r} + \frac{1}{2} \right) \quad (2.8)$$

We see that the mean energy \mathcal{E}_m decreases as the pump (η_r) increases — it is a well known results [1, 3–10].

B. Negative detuning

We consider the case of the negative detuning ($\Delta = -\omega_m$, laser frequency is *larger* than the cavity frequency) and start from the basic set of equations using the same notations as in (2.2) (see details in Appendix A):

$$(\gamma + i\nu) \mathbf{a}_{-} + iG_0 \mathbf{b}^{\dagger} = \sqrt{2\gamma} \mathbf{a}_{\text{in}-}, \quad (2.9a)$$

$$-iG_0^* \mathbf{a}_{-} + (\gamma_m + i\nu) \mathbf{b}^{\dagger} = \sqrt{2\gamma_m} \mathbf{b}_{\text{th}}^{\dagger} + f, \quad (2.9b)$$

$$\mathbf{a}_{\text{in}}[\gamma - i(2\omega_m + \nu)] = \sqrt{2\gamma} \mathbf{a}_{\text{in}}, \quad (2.9c)$$

$$\mathbf{a}_{\text{out}}(\pm\Omega) = -\mathbf{a}_{\text{in}}(\pm\Omega) + \sqrt{2\gamma} \mathbf{a}(\pm\Omega). \quad (2.9d)$$

Again the first two equations in (2.9) form the system of equations with the determinant:

$$D = (\gamma + i\nu)(\gamma_m + \eta + i\nu), \quad (2.10)$$

$$\eta = \frac{-|G_0|^2}{\gamma + i\nu} = \eta_r + i\eta_i, \quad \eta_r = \frac{-|G_0|^2\gamma}{\gamma^2 + \nu^2}, \quad (2.11)$$

Here η_r is a *negative* ponderomotive damping. We find \mathbf{a}_{-} and calculate \mathbf{a}_{out}

$$\mathbf{a}_{\text{out}-} = \frac{\gamma - i\nu}{\gamma + i\nu} \left(1 - \frac{2\eta_r}{\gamma_m + \eta + i\nu} \right) \mathbf{a}_{\text{in}-} \quad (2.12)$$

$$- \frac{iG_0\sqrt{2|\eta_r|}}{|G_0|(\gamma_m + \eta + i\nu)} \left(\sqrt{2\gamma_m} \mathbf{b}_{\text{th}}^{\dagger} + f \right),$$

$$\mathbf{a}_{\text{out}} = \frac{\gamma + i(2\omega_m + \nu)}{\gamma - i(2\omega_m + \nu)} \mathbf{a}_{\text{in}}. \quad (2.13)$$

The second term in round brackets in (2.12) describes back action via the light pressure force. Similarly to the

previous case, the double-sided spectral density S_y of the output amplitude quadrature y equals to:

$$S_y = \frac{1}{2} + \frac{2|\eta_r|\gamma_m(n_T + 1)}{|\gamma_m + \eta - i\nu|^2}. \quad (2.14)$$

So we see that back action terms $\sim \eta^2$ are completely compensated in (2.14).

The case $\Delta = -\omega_m$ corresponds to the negative damping ($\eta_r < 0$) and, hence, to the mechanical heating. That can be shown by the direct calculation of the mechanical oscillator's mean energy:

$$\mathcal{E}_m = \hbar\omega_m \left(\frac{\gamma_m n_T}{\gamma_m - |\eta_r|} + \frac{1}{2} \frac{(\gamma_m + |\eta_r|)}{(\gamma_m - |\eta_r|)} \right) \quad (2.15)$$

As we see it is a well known result — the mean energy \mathcal{E}_m increases with the $|\eta_r|$ [1, 3–10] due to introduction of the negative damping.

III. DISCUSSION

It is important that back action cancellation shown above does not provide the possibility to surpass SQL. To show it for the case of positive detuning we rewrite the spectral density S_y (2.7) so that it would have a form of the dimensionless force f_s defined in (2.2b):

$$S_f = \frac{(\gamma_m + \eta_r)^2 + \nu^2}{4\eta_r} + \gamma_m n_T \quad (3.1)$$

For the case of zero mechanical damping $\gamma_m = 0^1$ the detection condition of the resonant signal force $f_s = f_0 \cos \omega_m t$ is as follows

$$f_0^2 > \int_{\Delta\Omega} 2S_f(\Omega) \frac{d\Omega}{2\pi} \simeq \left(\frac{\eta_r}{2} + \frac{\Delta\Omega^2}{6\eta_r} \right) \frac{\Delta\Omega}{2\pi} \quad (3.2)$$

Optimizing this formula over η_r and putting $\Delta\Omega/2\pi \simeq 1/\tau$ (τ is the time throughout which the signal force acts and it is measurement time) we get the value of the minimal detectable force:

$$f_0 > \xi \frac{1}{\tau}, \quad \text{or } F_0 > \frac{\hbar f_0}{x_0} = \xi \frac{\sqrt{2\hbar m \omega_m}}{\tau}, \quad (3.3)$$

where ξ is a factor about 1. Obviously, formula (3.3) describes SQL [12, 13].

So we may conclude that, despite the fact that *fluctuation* back action is completely compensated, the *dynamical* back action (which corresponds to the introduced damping η_r) is responsible for the SQL restriction.

For the case of non-zero damping γ_m and narrow bandwidth ($\Delta\Omega < \gamma_m$ or $\gamma_m \tau > 1$) we have the minimum of S_f at $\eta_r = \gamma_m$ and the minimal force is equal to

$$F_0 > \sqrt{\frac{4\hbar m \omega_m \gamma_m (n_T + 1)}{\tau}}. \quad (3.4)$$

These formulas for the minimal signal force coincide with the usual one [12, 13]. We see that even for the case $n_T = 0$ restriction of thermal fluctuations does not vanish — in contrast to the formula (2.7) where at $n_T = 0$ the second term vanishes. It may also be explained by the *dynamical* back action.

For the case of negative detuning the introduced damping η_r is negative and in order to compensate possible instability feed back should be used. The detailed analysis shows that formulas (3.3) and (3.4) are still valid for the negative detuning case.

We emphasize that we used resolved sideband condition (2.1), that allows to make calculations simple and obvious. However, we also made accurate self-consistent calculations of spectral density S_y of the output amplitude quadrature in general using the set (A10) in Appendix A taking into account the interaction of both sidebands with the mechanical oscillator. We found that the back action cancellation takes place for the measurement of the *amplitude* quadrature in the output wave. However, if one measures the *phase* quadrature in the output wave the spectral density of the homodyne current contains back action terms ($\sim \eta_r^2$) but these terms are small enough. For example, for the case of positive detuning the formula (2.7) will contain an additional term, which may be estimated as:

$$(2.7): \quad S_y^{\text{add}} \simeq \frac{2\eta_r^2}{|\gamma_m + \eta - i\nu|^2} \times \frac{\gamma^2}{\omega_m^2}, \quad (3.5)$$

Obviously, the last multiplier is small due to the resolved sideband condition (2.1).

Appendix A: Details of model

In this Appendix we derive the main formulas. The electric fields E_{in} in the incident wave pump and the corresponding mean intensities J_{in} of light beam can be written as follows [20]:

$$E_{\text{in}} \simeq \sqrt{\frac{2\pi\hbar\omega_L}{S_c}} e^{-i\omega_L t} \times \quad (A1)$$

$$\times \left(A_{\text{in}} + \int_{-\infty}^{\infty} a_{\text{in}}(\Omega) e^{-i\Omega t} \frac{d\Omega}{2\pi} \right) + \{\text{h.c.}\},$$

$$J_{\text{in}} = \hbar\omega_L |A_{\text{in}}|^2, \quad (A2)$$

$$[a_{\text{in}}(\Omega), a_{\text{in}}^\dagger(\Omega')] = 2\pi \delta(\Omega - \Omega'), \quad (A3)$$

$$\langle a_{\text{in}}(\Omega) a_{\text{in}}^\dagger(\Omega') \rangle = 2\pi \delta(\Omega - \Omega'), \quad (A4)$$

$$\langle a_{\text{in}}^\dagger(\Omega) a_{\text{in}}(\Omega') \rangle = 0,$$

¹ For negative detuning the particular case of zero mechanical damping has no sense as ponderomotive negative damping creates instability.

where S is the cross section of the light beam, c is the velocity of light, a and a^\dagger are annihilation and creation operators.

For amplitude a_1 inside the cavity ($a_1 \equiv a_1(\Omega)$) and reflected amplitude a_{out} we have usual formulas

$$a_1 = \frac{\sqrt{\gamma/\tau} a_{\text{in}}}{\gamma - i(\Omega - \Delta)} + \frac{A_1 2ikx(\Omega)}{2\tau(\gamma - i(\Omega - \Delta))}, \quad (\text{A5a})$$

$$A_1 = \frac{A_{\text{in}} \sqrt{\gamma/\tau}}{\gamma + i\Delta}, \quad \gamma \equiv \frac{\Gamma^2}{4\tau}, \quad \tau \equiv \frac{L}{c}, \quad (\text{A5b})$$

$$a_{\text{out}} = -a_{\text{in}} + 2\sqrt{\gamma\tau} a_1 = -\frac{\gamma + i(\Omega - \Delta)}{\gamma - i(\Omega - \Delta)} a_{\text{in}} + \frac{iGx(\Omega)}{\gamma + i(\Delta - \Omega)}, \quad (\text{A5c})$$

$$G \equiv 2kA_1 \sqrt{\gamma/\tau}, \quad |G|^2 = \frac{4kI_0\gamma}{\hbar L}, \quad I_0 = \hbar\omega_0 |A_1|^2.$$

Writing an equation for the mechanical oscillator we take into account the light pressure force F_{pm} acting on the mechanical oscillator:

$$m(\omega_m^2 - 2i\gamma_m\Omega - \Omega^2)x(\Omega) = F_{\text{pm}} + F_{\text{th}} + F, \quad (\text{A6a})$$

$$F_{\text{pm}} = 2\hbar k A_1^* a_1(\Omega) + 2\hbar k A_1 a_1^\dagger(-\Omega). \quad (\text{A6b})$$

Here F_{pm} is a fluctuation light pressure force and F_{th} is a thermal force. We express displacement x through the annihilation and creation operators a_m , a_m^\dagger :

$$x(t) = x_0 (a_m(t) + a_m^\dagger(t)), \quad x_0 \equiv \sqrt{\frac{\hbar}{2m\omega_m}},$$

$$a_m(t) = \int_0^\infty [a_m(\Omega) e^{-i\Omega t}] \frac{d\Omega}{2\pi}, \quad (\text{A7a})$$

$$a_m^\dagger(t) = \int_0^\infty [a_m^\dagger(\Omega) e^{i\Omega t}] \frac{d\Omega}{2\pi}, \quad (\text{A7b})$$

and rewrite (A6a) for the Fourier transform of an annihilation operator a_m (below we denote $a_m \equiv a_m(\Omega)$)

$$a_m(i[\omega_m - \Omega] + \gamma_m) = \sqrt{2\gamma_m} b_{\text{th}} - \frac{x_0}{i\hbar} (F_{\text{pm}} + F), \quad (\text{A8a})$$

$$\langle b_{\text{th}}^\dagger(\Omega) b_{\text{th}}(\Omega') \rangle = 2\pi n_T \delta(\Omega - \Omega'), \quad (\text{A8b})$$

$$[b_{\text{th}}(\Omega), b_{\text{th}}^\dagger(\Omega')] = 2\pi \delta(\Omega - \Omega'). \quad (\text{A8c})$$

Here b_{th} is an operator describing thermal forces.

The set of equations. For a_1 , b_m , a_{out} (a_1 is the fluctuation amplitude of the wave inside the cavity) using (A5, A7) it is convenient to write (here we assume $\Omega > 0$ and denote $a_1 \equiv a_1(\Omega)$, $a_{1-} \equiv a_1(-\Omega)$)

$$a_1 [\gamma - i(\Omega - \Delta)] - \frac{A_1 2ikx_0}{2\tau} b_m = \sqrt{\frac{\gamma}{\tau}} a_{\text{in}}, \quad (\text{A9a})$$

$$a_{1-} [\gamma - i(\Omega + \Delta)] + \frac{A_1^* 2ikx_0}{2\tau} b_m = \sqrt{\frac{\gamma}{\tau}} a_{\text{in-}}, \quad (\text{A9b})$$

$$-2ikx_0 [A_1^* a_1 + A_1 a_{1-}^\dagger] + b_m [\gamma_m + i(\omega_m - \Omega)] = \sqrt{2\gamma_m} b_{\text{th}} + f_s, \quad (\text{A9c})$$

$$a_{\text{out}}(\pm\Omega) = -a_{\text{in}}(\pm\Omega) + 2\sqrt{\gamma\tau} a_1(\pm\Omega). \quad (\text{A9d})$$

We introduce notations in order to rewrite this set in the conventional form

$$a \equiv \sqrt{2\tau} a_1, \quad G_0 \equiv \frac{-A_1 2kx_0}{\sqrt{2\tau}}, \quad (\text{A10a})$$

$$|G_0| = \sqrt{\frac{kI_0}{mL\omega_m}}, \quad I_0 = \hbar\omega_0 |A_1|^2, \quad (\text{A10b})$$

$$a [\gamma - i(\Omega - \Delta)] + iG_0 b_m = \sqrt{2\gamma} a_{\text{in}}(\Omega), \quad (\text{A10c})$$

$$a_{1-} [\gamma - i(\Omega + \Delta)] - iG_0^* b_m = \sqrt{2\gamma} a_{\text{in-}}, \quad (\text{A10d})$$

$$i[G_0^* a + G_0 a_{1-}^\dagger] + b_m [\gamma_m + i(\omega_m - \Omega)] = \sqrt{2\gamma_m} b_{\text{th}} + f_s. \quad (\text{A10e})$$

$$a_{\text{out}}(\pm\Omega) = -a_{\text{in}}(\pm\Omega) + 2\sqrt{\gamma\tau} a_1(\pm\Omega). \quad (\text{A10f})$$

For the case of positive detuning $\Delta = \omega_m$ we make an approximation using the condition (2.1): we omit the term $(-iG_0^* b_m)$ in the Eq. (A10d) and the term $(iG_0 a_{1-}^\dagger)$ in (A10e). As a result we obtain the set (2.2)

For the case of negative detuning $\Delta = -\omega_m$ we take the set that is complex conjugated to (A10) and making similar approximations we get (2.9).

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